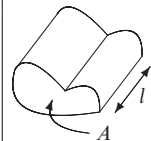


7.6 LCR circuits

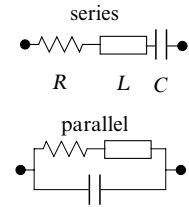
LCR definitions

Current	$I = \frac{dQ}{dt}$	(7.139)	I current Q charge
Ohm's law	$V = IR$	(7.140)	R resistance V potential difference over R I current through R
Ohm's law (field form)	$\mathbf{J} = \sigma \mathbf{E}$	(7.141)	$\mathbf{J}$ current density $\mathbf{E}$ electric field σ conductivity
Resistivity	$\rho = \frac{1}{\sigma} = \frac{RA}{l}$	(7.142)	ρ resistivity A area of face (I is normal to face) l length
Capacitance	$C = \frac{Q}{V}$	(7.143)	C capacitance V potential difference across C
Current through capacitor	$I = C \frac{dV}{dt}$	(7.144)	I current through C t time
Self-inductance	$L = \frac{\Phi}{I}$	(7.145)	Φ total linked flux I current through inductor
Voltage across inductor	$V = -L \frac{dI}{dt}$	(7.146)	V potential difference over L
Mutual inductance	$L_{12} = \frac{\Phi_1}{I_2} = L_{21}$	(7.147)	Φ_1 total flux from loop 2 linked by loop 1 L_{12} mutual inductance I_2 current through loop 2
Coefficient of coupling	$ L_{12} = k \sqrt{L_1 L_2}$	(7.148)	k coupling coefficient between L_1 and L_2 (≤ 1)
Linked magnetic flux through a coil	$\Phi = N\phi$	(7.149)	Φ linked flux N number of turns around ϕ ϕ flux through area of turns



Resonant LCR circuits

Phase resonant frequency ^a	$\omega_0^2 = \begin{cases} 1/LC & \text{(series)} \\ 1/LC - R^2/L^2 & \text{(parallel)} \end{cases}$ (7.150)	ω_0 resonant angular frequency L inductance C capacitance R resistance
Tuning ^b	$\frac{\delta\omega}{\omega_0} = \frac{1}{Q} = \frac{R}{\omega_0 L}$ (7.151)	$\delta\omega$ half-power bandwidth Q quality factor
Quality factor	$Q = 2\pi \frac{\text{stored energy}}{\text{energy lost per cycle}}$ (7.152)	



^aAt which the impedance is purely real.

^bAssuming the capacitor is purely reactive. If L and R are parallel, then $1/Q = \omega_0 L/R$.

Energy in capacitors, inductors, and resistors

Energy stored in a capacitor	$U = \frac{1}{2} C V^2 = \frac{1}{2} Q V = \frac{1}{2} \frac{Q^2}{C}$ (7.153)	U stored energy C capacitance Q charge V potential difference
Energy stored in an inductor	$U = \frac{1}{2} L I^2 = \frac{1}{2} \Phi I = \frac{1}{2} \frac{\Phi^2}{L}$ (7.154)	L inductance Φ linked magnetic flux I current
Power dissipated in a resistor ^a (Joule's law)	$W = I V = I^2 R = \frac{V^2}{R}$ (7.155)	W power dissipated R resistance
Relaxation time	$\tau = \frac{\epsilon_0 \epsilon_r}{\sigma}$ (7.156)	τ relaxation time ϵ_r relative permittivity σ conductivity

^aThis is d.c., or instantaneous a.c., power.

Electrical impedance

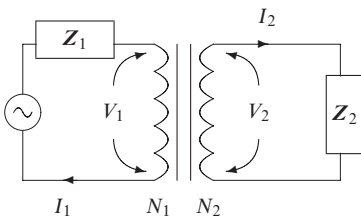
Impedances in series	$Z_{\text{tot}} = \sum_n Z_n$ (7.157)
Impedances in parallel	$Z_{\text{tot}} = \left(\sum_n Z_n^{-1} \right)^{-1}$ (7.158)
Impedance of capacitance	$Z_C = -\frac{j}{\omega C}$ (7.159)
Impedance of inductance	$Z_L = j\omega L$ (7.160)
Impedance: Z Inductance: L Conductance: $G = 1/R$ Admittance: $Y = 1/Z$	Capacitance: C Resistance: $R = \text{Re}[Z]$ Reactance: $X = \text{Im}[Z]$ Susceptance: $S = 1/X$



Kirchhoff's laws

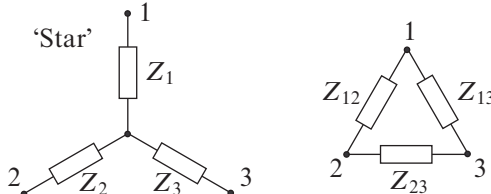
Current law	$\sum_{\text{node}} I_i = 0$	(7.161)	I_i currents impinging on node
Voltage law	$\sum_{\text{loop}} V_i = 0$	(7.162)	

Transformers^a

	n turns ratio N_1 number of primary turns N_2 number of secondary turns V_1 primary voltage V_2 secondary voltage I_1 primary current I_2 secondary current Z_{out} output impedance Z_{in} input impedance Z_1 source impedance Z_2 load impedance
Turns ratio	$n = N_2/N_1$ (7.163)
Transformation of voltage and current	$V_2 = nV_1$ (7.164)
	$I_2 = I_1/n$ (7.165)
Output impedance (seen by Z_2)	$Z_{\text{out}} = n^2 Z_1$ (7.166)
Input impedance (seen by Z_1)	$Z_{\text{in}} = Z_2/n^2$ (7.167)

^aIdeal, with a coupling constant of 1 between loss-free windings.

Star-delta transformation

		i, j, k node indices (1, 2, or 3) Z_i impedance on node i Z_{ij} impedance connecting nodes i and j
Star impedances	$Z_i = \frac{Z_{ij}Z_{ik}}{Z_{ij} + Z_{ik} + Z_{jk}}$	(7.168)
Delta impedances	$Z_{ij} = Z_i Z_j \left(\frac{1}{Z_i} + \frac{1}{Z_j} + \frac{1}{Z_k} \right)$	(7.169)

